

**Math Kangaroo 2002**

**Level of grades 11 - 12**

**Problems 3 points each**

1. A certain polyhedron has exactly  $n$  faces and one of these faces is a pentagon. What is the least possible value of  $n$ ?

- A) 5                      B) 6                      C) 7                      D) 8                      E) 10

2. With how many zeros does the product of the first 2002 prime numbers in standard notation end?

- A) 0                      B) 1                      C) 10                      D) 20                      E) 100

3. A certain prism has 2002 vertices. How many edges does this prism have?

- A) 3003                      B) 1001                      C) 2002                      D) 4002                      E) 2001

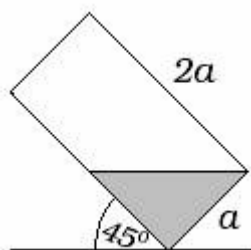
4. The smallest natural number that has exactly 7 divisors is equal to

- A) 24                      B) 48                      C) 60                      D) 64                      E)  $3^6$

5. The triangle with sides 6 and 8 has the largest area when its third side is equal to

- A) 6                      B) 7                      C) 8                      D) 10                      E) 12

6. The figure below shows the axial cross-section of a glass with liquid, shaped like a cylinder with base's diameter  $a$  and height  $2a$ . The angle between the surface and the glass is  $45^\circ$ . What part of the glass's volume does the fluid take?



- A) Less than  $\frac{1}{6}$                       B)  $\frac{1}{6}$                       C)  $\frac{1}{4}$                       D)  $\frac{1}{3}$                       E) More than  $\frac{1}{3}$

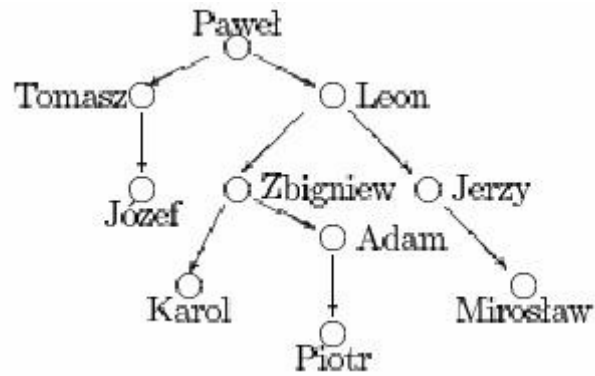
7. For what positive integer  $n$  will the distance between numbers  $2^0 + 2^1 + 2^2 + \dots + 2^n$  and 2002 be the smallest?

- A) 7                      B) 8                      C) 9                      D) 10                      E) 11

8. In a certain hotel on average 88% of rooms are occupied during the three summer months and on average 64% of rooms are occupied during the other nine months. What is the average percent of rooms occupied during the whole year in that hotel?

- A) 108%                      B) 54%                      C) 70%                      D) 64%                      E) 88%

9. Piotr created a genealogical tree of the men in his family. The arrows go from a father to a son. What was the



name of the son of the brother of the grandfather of Piotr's father's brother?

- A) Józef      B) Mirosław      C) Leon      D) Zbigniew      E) other answer

10. How many spheres with radius 1 have the total volume equal to the volume of a sphere with radius 2?

- A) 2      B) 4      C) 6      D) 8      E) 16

**Problems 4 points each**

11. When it freezes water increases its volume by  $\frac{1}{10}$ . By how much does the volume of melted ice decrease?

- A) By  $\frac{1}{12}$       B) By  $\frac{1}{10}$       C) By  $\frac{1}{11}$       D) By  $\frac{1}{13}$       E) By  $\frac{1}{9}$

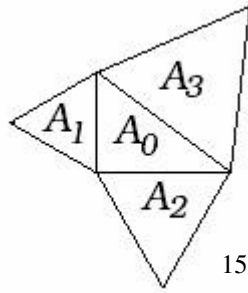
12. Let  $a$  and  $b$  be natural numbers. If the greatest common factor of numbers  $a$  and  $b$  is equal to 3 and  $\frac{a}{b} = 0.4$ , then the product  $ab$  equals

- A) 18      B) 10      C) 36      D) 30      E) 90

13. The approximate length of the equator of the Earth is 40,000 km. The approximate length of the parallel latitude of  $60^\circ$  is equal to

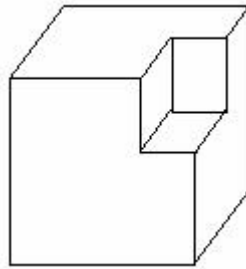
- A) 34,000 km      B) 23,500 km      C) 26,700 km      D) 30,000 km      E) 20,000 km

14. The figure below shows four triangles with areas  $A_0, A_1, A_2, A_3$ . The triangle with area  $A_0$  is a right triangle and the others are equilateral. Which of the equations below is true?



- A)  $A_1 + A_2 = A_3$
- B)  $(A_1)^2 + (A_2)^2 = (A_3)^2$
- C)  $A_1 + A_2 + A_3 = 3A_0$
- D)  $A_1 + A_2 = \sqrt{2} A_3$
- E)  $A_1 + A_2 = \sqrt{3} A_3$

15. Achilles chases the turtle that is running away from him. At the very beginning the distance between them was 990 m. Achilles runs 10 m in every second and the turtle runs 1 m in every 10 seconds. How long will it take Achilles to catch the turtle?
- A) 1 min 40 sec
  - B) 990 sec
  - C) 1 min 39 sec
  - D) 1 min 50 sec
  - E) He will never catch the turtle.



16. From a stone cube with the volume of  $512 \text{ dm}^3$  a small rectangular solid was cut off, as you can see in the picture. What is the total surface area of the remaining solid?
- A)  $320 \text{ dm}^2$
  - B)  $336 \text{ dm}^2$
  - C)  $384 \text{ dm}^2$
  - D)  $468 \text{ dm}^2$
  - E) It cannot be determined from the information given.

17. If  $\log_a x^2 = 1$ ,  $\log_a \sqrt{y} = 2$ , then  $\log_a(\sqrt{x} y^2)$  is equal to

- A) 4
- B) 7
- C) 17
- D) 33
- E)  $\frac{33}{4}$

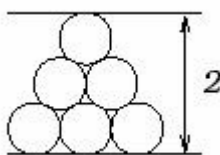
18. Ten teams took part in a soccer tournament. In each match the winning team received 3 points, the losing team received 0 points and for a tie both teams received one point. Each team played with every other team exactly one match. In total, all of the teams received 130 points. How many matches ended with a tie?

- A) 1
- B) 2
- C) 3
- D) 4
- E) 5

19. On a plane, set  $A$  of ten points is given. Exactly five of the points lie on one line and no other line passes through more than two of those ten points. How many triangles can be made with vertices that belong to set  $A$ ?

- A) 20
- B) 50
- C) 70
- D) 100
- E) 110

20. Each of the circles in the picture has the same length radius. What is it?



- A)  $\frac{1}{1 + \sqrt{3}}$
- B)  $\frac{2}{1 + \sqrt{3}}$
- C)  $\frac{3}{2 + \sqrt{3}}$
- D)  $\frac{1}{2 + \sqrt{3}}$
- E) Other number

**Problems 5 points each**

21. While fishing, Pawel caught as many fish as his son Mirek. Zbyszek caught three times as much fish as his son. Altogether, they caught 35 fish. What's the name of Zbyszek's son?

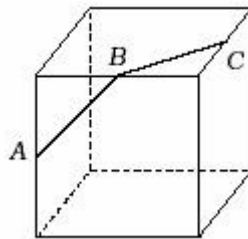
- A) The described situation is impossible.    B) Zbyszek    C) Pawel D) Mirek  
E) It cannot be determined from the information given.

22. The last digit of the number  $2001^{2001}$  is of course 1. What is the digit before the last digit of that number?

- A) 9                      B) 6                      C) 5                      D) 1                      E) 0

23. In the inequalities below the value of the angle is expressed in radian measure. Which of the inequalities below is true?

- A)  $\sin 1 < \sin 2 < \sin 3$                       B)  $\sin 3 < \sin 2 < \sin 1$                       C)  $\sin 1 < \sin 3 < \sin 2$   
D)  $\sin 2 < \sin 1 < \sin 3$                       E)  $\sin 3 < \sin 1 < \sin 2$



24.

What is the measure of the angle between segments  $AB$  and  $BC$  if  $A, B, C$  are the midpoints of the cube's edges (see the picture)?

- A)  $90^\circ$                       B)  $100^\circ$                       C)  $110^\circ$                       D)  $120^\circ$                       E)  $135^\circ$

25. The polynomial  $f(x)$  satisfies the condition  $f(x + 1) = x^2 + 4x$ . What is  $f(x - 1)$  equal to?

- A)  $x^2 - 4$                       B)  $x^2 - 4x$                       C)  $4x - x^2$                       D)  $x^2 + 2x - 3$                       E)  $x^2 + 4x$

26. There is a certain sequence of positive real numbers. Beginning from the third term each term of the sequence is the sum of all the previous terms. The seventh term is equal to 1000 and the first term is equal to 1. The second term of this sequence is equal to

- A) 2                      B)  $\frac{123}{2}$                       C)  $\frac{121}{4}$                       D) 124                      E) Other answer

27. How many no-congruent triangles are there that are made by vertices of a regular octagon.

- A) 2                      B) 5                      C) 6                      D) 8                      E) 10

28. Let  $a, b, c$  be real numbers.

If  $a + b + c = 7$ ,  $\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} = \frac{7}{10}$

then  $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = ?$

- A)  $\frac{19}{10}$       B)  $\frac{17}{10}$       C)  $\frac{9}{7}$       D)  $\frac{3}{2}$       E)  $\frac{10}{7}$

29. How many positive integers from 1 to  $10^{2002}$  have the sum of their digits equal to 2?

A) 2,007,006      B) 2,005,003      C) 2,003,001      D) 2,005,002      E) Other answer

30. In a certain strange language, words are written with letters from the following six-letter alphabet:  $A, G, K, N, R, U$ . Each word consists of six letters and none of the letters repeats. Each combination of these six letters is a word in this language. Which entry in the dictionary is the word “*KANGUR*” ?

A) 265<sup>th</sup>      B) 253<sup>rd</sup>      C) 246<sup>th</sup>      D) 248<sup>th</sup>      E) Other answer

[Back to all problems](#)