

Math Kangaroo 2003
Level of grades 11 - 12

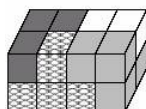
30. The figure shows a parabola $y = ax^2 + bx + c$, where coefficients a, b, c are all different, and a line. The x -intercept and y -intercept of the line are the common points of the parabola and axes. Which of the following equations represents this line?

- Problems 3 points each**
 A) $y = bx + c$ B) $y = cx + b$ C) $y = ax + b$
 D) $y = ax + c$ E) $y = cx + a$

1. There are 100 roses growing in Aunt Agata's garden. Each rose is either red or yellow. It is known that at least one rose is yellow, and among every two roses at least one is red. How many yellow roses are there in Aunt Agata's garden?

- A) 50 B) 49 C) 51 D) 99 E) 1

2. The rectangular prism, shown in the picture, was made out of four blocks. Each block consists of four small cubes. Which one from the blocks below has the same shape as the white block in the picture?



3. In a train to Warsaw Ania sat in the dining car, which was the seventh, counting from the front of the train, and Tomek sat in the car, which was the sixth from the end of the train and was located in front of the dining car. How many cars did this train consist of?

- A) 14 B) 13 C) 12 D) Less than 12 E) Nothing can be said about the number of cars.

4. The area of the square shown in the picture equals a . The area of each circle equals b . What is the area of the figure circled with the bold line?



- A) $3b$ B) $2a + b$ C) $a + 2b$ D) $3a$ E) $a + b$

5. Calculating the volume of a sphere, Leon, by mistake, put the length of the diameter into the formula for volume, instead of the radius. What should he do in order to get the correct result?

- A) Divide by 2 B) Divide by 4 C) Divide by 6 D) Divide by 8 E) Divide by 16

6. If n is a natural number, then $2^{n+2003} + 2^{n+2003}$ is equal to:

- A) 2^{n+2004} B) 2^{n+4006} C) $2^{2n+4006}$ D) $2^{2n+2003}$ E) 2^{n+2003}

7. The average number of students admitted to school within the four-year period of 1998-2001 was equal to 325 per year. The average number of students admitted to school within the five-year period of 1998-2002 was 20% more than the number of students admitted in the four-year period. How many students were admitted in 2002?

- A) 650 B) 600 C) 455 D) 390 E) 345

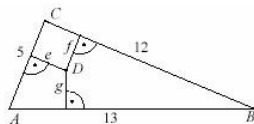
8. For any positive integers $a, b, m,$ and $n,$ satisfying the condition $\frac{m}{n} = \frac{a}{a+b}$, the difference $\frac{a}{b} - \frac{m}{n}$ is equal to

- A) 0 B) $\frac{nb}{ma}$ C) $\frac{mb}{na}$ D) $\frac{ma}{nb}$ E) $\frac{ab}{mn}$

9. The numbers 15, 13, and 12 are the lengths of two sides of an obtuse triangle and the altitude perpendicular to the third side, respectively. What is the area of this triangle?
- A) 168 B) 80 C) 84 D) $6\sqrt{65}$ E) It is impossible to solve.
10. The number $a = 111\dots\dots 11$ consists of 2003 digits of 1. What is the sum of the digits of the number $2003 \times a$?
- A) 10,000 B) 10,015 C) 10,020 D) 10,030 E) 2003×2003

Problems 4 points each

11. The area of triangle ABC is equal to 30. Let D be any internal point of this triangle and let e , f , and g indicate the distances between the point D and the sides of the triangle. What is the sum of $5e + 12f + 13g$?



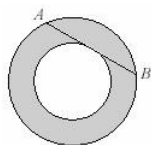
- A) 120 B) 90 C) 60 D) 30 E) This depends on the location of point D .
12. Into the rectangles, which create the figure shown below at the right, natural numbers greater than 1 are written as it is shown below at the left. Which of the numbers below cannot be placed in the shaded rectangle?



- A) 60 B) 88 C) 90 D) 100 E) 145
13. Given the data below, which of the following could be the elements of triangle ABC ?
- A) $AB = 11$ cm, $BC = 19$ cm, $CA = 7$ cm
 B) $AB = 11$ cm, $BC = 6$ cm, $\angle BAC = 60^\circ$
 C) $AB = 11$ cm, $CA = 7$ cm, $\angle CBA = 128^\circ$
 D) $AB = 11$ cm, $\angle BAC = 63^\circ$, $\angle CBA = 128^\circ$
 E) $AB = 11$ cm, $BC = 10$ cm, $\angle BAC = 60^\circ$
14. There is a cube with the edge of 1 and eight spheres with radii of $\frac{1}{2}$ and their centers located at the vertices of the cube. What is the diameter of the sphere that is outside tangent to the eight given spheres?

- A) $\sqrt{3} - 1$ B) $\frac{1}{2}$ C) $\sqrt{2} - 1$ D) $\frac{\sqrt{3}}{4}$ E) That sphere does not exist.

15. Two circles seen in the figure are concentric. Chord AB of the larger circle is tangent to the smaller circle and its length is equal to 16. What is the area of the shaded region?



- A) 32π B) 63π C) 64π D) $32\pi^2$ E) $16^2 - 16\pi$

16. How many terms of the sequence, which terms are the seventh powers of consecutive natural numbers, are located between the numbers: $5^{21} + 1$ and $2^{49} - 1$?

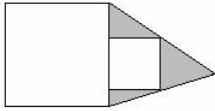
- A) 6 B) 5 C) 4 D) 3 E) 2

17. The set of all values of parameter m , for which the circle: $x^2 + y^2 = 1$ and the parabola $y = x^2 + m$ have exactly one common point, is:

- A) $\{-\frac{5}{4}, -1, 1\}$ B) $\{-\frac{5}{4}, 1\}$ C) $\{-1, 1\}$ D) $\{-\frac{5}{4}\}$ E) $\{1\}$

18. The larger square has a side of 2 m, the smaller square has a side of 1m. What is the area of the shaded region

(see the figure)?



- A) 1 m^2 B) 2 m^2 C) $2\sqrt{2} \text{ m}^2$ D) 4 m^2 E) The area depends on the location of the smaller square.

19. The value of expression : $100^2 - 99^2 + 98^2 - 97^2 + \dots + 2^2 - 1^2$ equals:

- A) 2002 B) 2020 C) 4040 D) 5050 E) 8008

20. If $(a + \frac{1}{a})^2 = 6$ and $a > 0$, then $a^3 + \frac{1}{a^3}$ is equal to:

- A) $4\sqrt{6}$ B) $3\sqrt{6}$ C) 6 D) $5\sqrt{6}$ E) $6\sqrt{6}$

Problems 3 points each

21. The value of the expression: $\sqrt{\sqrt{\sqrt{\sqrt{1+2000\sqrt{1+2001\sqrt{1+2002\sqrt{1+2003 \times 2005}}}}}}}$ is equal to :

- A) 2000 B) 2001 C) 2002 D) 2003 E) 2004

22. The remainder of division of polynomial $x^{2003} + x^2 + x + 5$ by $x^2 - 1$ is equal to:

- A) $x + 1$ B) $2x + 6$ C) $x - 1$ D) 4 E) $x + 5$

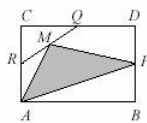
23. Let a and b be such prime numbers that $a > b$ and numbers $a - b$ and $a + b$ are also prime numbers. Then the number $S = a + b + (a - b) + (a + b)$ is:

- A) An even number. B) Divisible by 3. C) Divisible by 5. D) Divisible by 7. E) A prime number.

24. How many factors of number $2^{12} - 1$ are there between 10 and 100?

- A) 9 B) 7 C) 10 D) 5 E) 4

25. In rectangle $ABDC$ points P , Q , and R are respectively the midpoints of sides BD , CD , and CA . Let M be the midpoint of the segment QR . The ratio of the area of triangle APM to the area of rectangle $ABDC$ is:



- A) $\frac{1}{4}$ B) $\frac{1}{6}$ C) $\frac{3}{8}$ D) $\frac{1}{3}$ E) $\frac{5}{16}$

26. The sequence $\{a_n\}$, $n \geq 0$, is defined in the following way: $a_0 = 4$, $a_1 = 6$, $a_{n+1} = \frac{a_n}{a_{n-1}}$ ($n \geq 1$). Then a_{2003} is:

- A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) 4 D) $\frac{1}{4}$ E) $\frac{1}{6}$

27. Point P is located inside a circle with radius 15. The distance from this point to the center of the circle is 9. How many different chords exist through point P and such that their lengths are expressed with whole numbers?

- A) 1 B) 2 C) 7 D) 12 E) 24

28. Let $\{a_n\}$ be an arithmetic sequence that is not constant and $\{b_n\}$ a geometric sequence that is not constant. Let assume that $a_{40} = b_{40} > 0$ and $a_{60} = b_{60} > 0$. Then:

- A) $a_{50} = b_{50}$ B) $a_{50} < b_{50}$ C) $a_{50} > b_{50}$ D) $a_{40} = b_{60}$ E) $a_{60} = b_{40}$

28. Six points were chosen on a circle and every possible chord with end points in those points was drawn. Two chords, which do not have the common points are named separate. How many pairs of separate chords does exist in the situation described above?

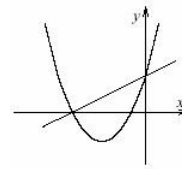
A) 26

B) 28

C) 30

D) 34

E) 36



29. The figure show parabola $y = ax^2 + bx + c$ where parameters a, b, c

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