

MATH KANGAROO 2004 in USA
Level of Grades 9 - 10

A ——— ~~B~~

3 points each

1. The value of the expression $(1 - 2) - (3 - 4) - (5 - 6) - \dots - (99 - 100)$ is equal to:

- A) 0 B) 49 C) - 48 D) 48 E) 50

2. Andrzej has a collection of 2004 cars. Half of them are blue, one fourth are red, and one sixth are green. How many cars of other color are there?

- A) 167 B) 334 C) 501 D) 1001 E) 1837

3. A pyramid consists of seven sides. How many edges does the pyramid have?

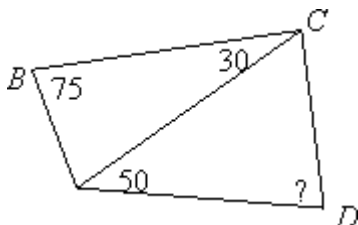
- A) 8 B) 9 C) 12 D) 18 E) 21

4. A swimming pool has the shape of a rectangle with dimensions 40m x 60m. The perimeter of this rectangle shown on a plan is 100 cm. What is the scale of this plan?

- A) 1 : 100 B) 1 : 150 C) 1 : 160 D) 1 : 170 E) 1 : 200

5. Andrzej and Milena have a certain number of coins. Andrzej received five additional coins from his grandfather and at that point he had twice as many as Milena. If he gave 12 coins to his grandmother he would have half as many as Milena. How many coins did Andrzej have at the beginning?

- A) 5 B) 7 C) 9 D) 11 E) 45



6. The measures of some of the angles of quadrilateral $ABCD$ are shown in the picture. What is the measure of angle ADC if $|BC| = |AD|$?

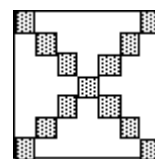
- A) 30° B) 50° C) 55° D) 65° E) 70°

A

7. There are 30 mushrooms in a basket, some are white and some brown. There is at least one brown mushroom among 12 randomly picked mushrooms, and at least one white mushroom among 20 randomly picked mushrooms. How many white mushrooms are there in the basket?

- A) 11 B) 12 C) 19 D) 20 E) 21

8. A square with a side equal to 2003 was divided into small equal squares each with side of 1. Squares along the diagonals were shaded. The picture shows an example of shading a square with side of 7. What is the area of the region that is not shaded?



- A) 2002^2 B) $2002 \cdot 2001$ C) 2003^2 D) $2003 \cdot 2004$ E) 2004^2

9. A dartboard consists of a black circle and two rings: grey and white. The radius of the black circle is r and the width of each ring is r . How many times is the area of the grey ring greater than the area of the black circle?



- A) 2 times B) 3 times C) 4 times D) 5 times E) 6 times

10. Two real numbers a and b have different signs. Which of the numbers below is the greatest?

- A) $|a^2 - b^2|$ B) $(|a| - |b|)^2$ C) $(a - b)^2$ D) $(a + b)^2$ E) $a^2 + b^2$

4 points each

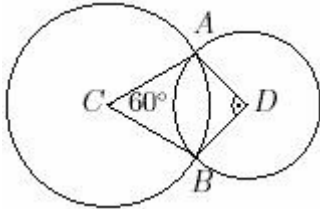
11. 770 nuts were divided between three girls in proportion to their age. For every three nuts taken by Ania, Milena took four nuts, and for every seven nuts taken by Natalia, Milena took six nuts. How many nuts did the youngest girl receive?

- A) 264 B) 256 C) 258 D) 198 E) 180

12. Each of five students wrote one number from the set $\{1, 2, 4\}$. Afterwards, the product of the written numbers was created. Which of the numbers below could be the result of this multiplication?

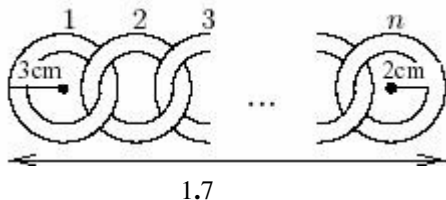
- A) 100 B) 256 C) 768 D) 2048 E) 4096

13. Two circles, with centers C and D , intersect at points A and B (see the picture). What is the ratio of the radii of those circles if angle $ACB = 60^\circ$ and angle $ADB = 90^\circ$?



- A) 4:3 B) $\sqrt{2} : 1$ C) 3:2 D) $\sqrt{3} : 1$ E) 2:1

14. Rings, with dimensions shown in the picture were linked together to create a chain 1.7 m long. How many rings were used to create that chain?



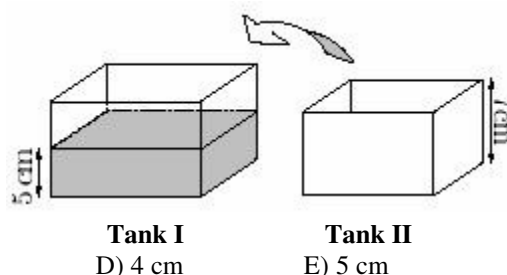
- A) 17 B) 21 C) 30 D) 42 E) 85

15. An hour hand of a clock is 4 cm long, and a minute hand is 8 cm long. What is the ratio of distances traveled by the ends of these hands during 3 hours?

- A) 1:2 B) 1:4 C) 1:6 D) 1:12 E) 1:24

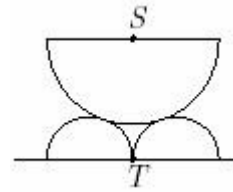
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16. The area of the base of tank I is equal to 2 dm^2 . The water in this tank is 5 cm high. The empty tank II with the area of its base equal to 1 dm^2 and height equal to 7 cm is put into tank I. The height of the water in tank I rose and some of the water overflowed into tank II. What is the depth of the water in tank II?



- A) 1 cm B) 2 cm C) 3 cm D) 4 cm E) 5 cm

17. Three semicircles, two of them with diameters equal to 2 dm and one with diameter of 4 dm, are placed as shown in the figure. What is the distance from center S of the bigger semicircle to point T ?



- A) 3 B) $\sqrt{8}$ C) 2.85 D) $\sqrt{10}$ E) 2.5

18. Andrzej places one number from the set $\{1, 2, 3, 4\}$ in each square of the diagram in such a way that each row and each column contains every number. In how many different ways can Andrzej finish filling the diagram, if he started filling out the diagram as shown in the picture?

- A) 1 B) 2 C) 4 D) 16 E) 128

1			
2	1		
	3		
	4		

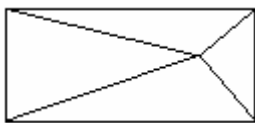
19. A 38-sided regular polygon was inscribed in a circle. The consecutive vertices of the polygon were numbered from 1 to 38. If vertex number 8 lies on one end of the circle's diameter, then the other end of the diameter lies on vertex number:

- A) 24 B) 25 C) 26 D) 27 E) 28

20. For how many natural numbers between 100 and 200 their prime factors are numbers from the set $\{2, 3\}$ only?

- A) 1 B) 3 C) 4 D) 5 E) 6

5 points each



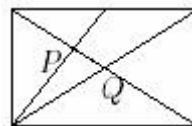
21. A rectangle was divided into four triangles with one common vertex. The bases of these triangles are the sides of the rectangle (see the figure). Which set of the numbers below may express the areas of these triangles?

- A) 4, 5, 8, 9 B) 3, 5, 6, 7 C) 5, 6, 7, 12 D) 10, 11, 12, 19 E) 5, 6, 8, 10

22. A test consists of 20 questions. For a correct answer a student gets 7 points, for every incorrect answer this student loses 2 points, and for no answer he gets 0 points. Andrzej received 87 points on his test. How many questions did he not answer?

- A) 2 B) 3 C) 4 D) 5 E) 6

23. The diagonals of a rectangle intersect at point Q . A segment connecting a vertex of a rectangle with the midpoint of the side of the rectangle crosses one of the diagonals at point P (see the figure). What is the ratio of the length of the diagonal to the

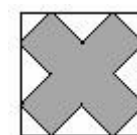


length of segment PQ ?

- A) It depends on the rectangle dimensions. B) 6 C) $\frac{13}{3}$ D) 4 E) 3

24. A picture shows a square and a shaded dodecagon. The sides of the dodecagon have the same length and are perpendicular to each other. The perimeter of the dodecagon is 36. What is the area of the square?

- A) 48 B) 72 C) 108 D) 115.2 E) 144



25. Out of all 3-digit natural numbers n smaller than 200, how many have the property that the number $(n+1)(n+2)(n+3)$ is divisible by 7?

- A) 42 B) 38 C) 34 D) 28 E) 16

26. There is a numerical sequence that consists of 200 zeros. The sequence is transformed into a different sequence of 200

terms in the following way: At the beginning we add 1 to each term of the sequence. In the second step we add 1 to all terms numbered with an even number in the sequence. In the third step we add 1 to all terms numbered with the number that is divisible by three, and so on. After 200 steps, we obtain a sequence with the 120th term equal to:

- A) 16 B) 12 C) 20 D) 24 E) 32

27. How many 8-digit numbers: $a_1a_2a_3 \dots a_8$, which consist of zeros or ones only ($a_1=1$), have such a property that $a_1+a_3+a_5+a_7 = a_2+a_4+a_6+a_8$?

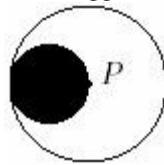
- A) 2^7 B) 35 C) 49 D) 16 E) 32

28. The area of the shaded region of the figure shown in the picture is equal to $2p$. What is the length of chord AB ?








- A) 1 B) 2 C) 3 D) 4 E) The length cannot be determined.

29. The picture shows two tangent circles with a ratio of radii equal to 1:2. The dark circle is rolled along the circumference of the bigger circle.



What is the shape of the path made by point P ?

- A)  B)  C)  D)  E) 

30. All numbers that are neither divisible by 5 nor 11 were removed from a sequence of consecutive natural numbers from 1 to 10,000. A new sequence was formed. What is the value of the 2004th term of this sequence?

- A) 1000 B) 5000 C) 10,000 D) 6545 E) 7348

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