Kangourou Sans Frontières **Kangourou Maths 2009** **Level:11-12**

(for students of grades 11 and 12 at Lyceum)

3 point questions:

1) There are 200 kangaroo. 1% of them have black eyes and the rest brown. How many kangaroo we have to remove so that the kangaroo with black eyes become 2%?

A) 2 **B)** 4 **C)** 20 **D)** 50 **E)** 100

2) Which one of the following number is the largest?

A) $\sqrt{2}-\sqrt{1}$ **B)** $\sqrt{3}-\sqrt{2}$ **C)** $\sqrt{4}-\sqrt{3}$ **D)** $\sqrt{5}-\sqrt{4}$ **E)** $\sqrt{6}-\sqrt{5}$

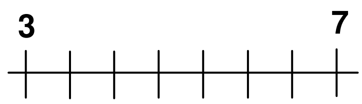
3) How many different natural numbers $n \geq 1$ are there with the property that n^2+n is a prime number?

A) none **B)** 1 **C)** 2 **D)** more than two and less than thousand **E)** infinitely many

4) Anna, Vasile and John entered a bookshop. Each one of them bought 6 books, 1 CD of classic music and 3 set of exercise books. The books were all of the same price. All CDs were also of the same price and all the exercise book sets were of the same price between them. The cost of every object was an integer value of euro. Which of the following costs could be the total cost that all three of them paid together (assuming that the remaining four values could not be the cost)?

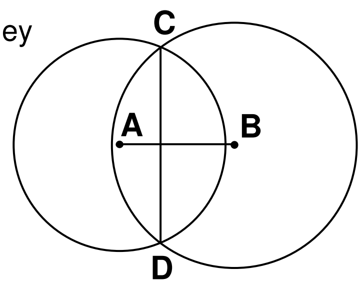
A) 392 ευρώ **B)** 382 ευρώ **C)** 372 ευρώ **D)** 362 ευρώ **E)** 352 ευρώ

5) In a line we write 8 numbers (not necessarily in a increasing order). In the figure we can see the first and the last number, while the ones in between are invisible. If the sum of two consecutive numbers in each line is the same in all cases, then what is the sum of the six invisible numbers?



A) 18 **B)** 42 **C)** 40 **D)** 30 **E)** we cannot determine it

6) Two circles with centres A and B have radii 13 and 15 respectively. If they intersect at C and D and if CD=24 what is the length of AB;

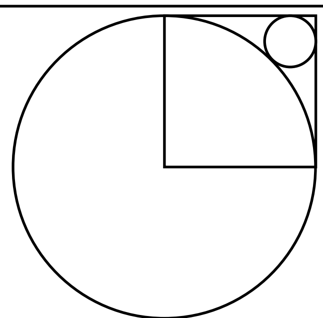


A) 2 **B)** 5 **C)** 9 **D)** 14 **E)** 18

7) A box contains 2 white, 3 red and 4 blue socks. Liz knows, that a third of the socks have a hole in them but not what colour the worn through socks are. She is picking up socks from the box to the floor at random and hoping to get two good socks of the same colour. How many socks must she take to be absolutely sure to have a good pair?

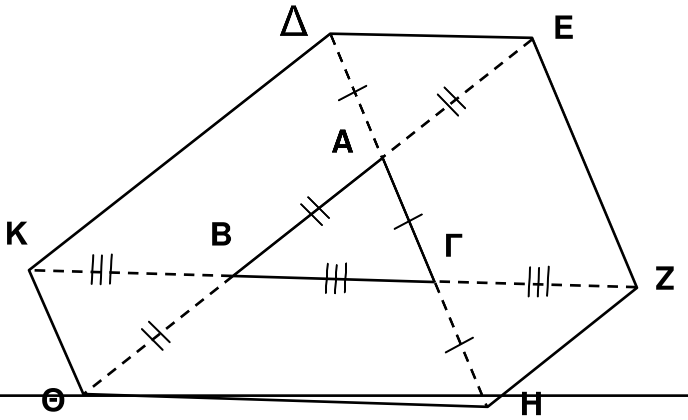
A) 2 **B)** 3 **C)** 6 **D)** 7 **E)** 8

8) The square in the figure has side length 1. The large circle has its centre at one of the vertices of the square with radius 1. The small circle is tangent to the square and the large circle. What is the value of the radius of small circle?



A) $\sqrt{2}-1$ **B)** $\frac{1}{4}$ **C)** $\frac{\sqrt{2}}{4}$ **D)** $1-\frac{\sqrt{2}}{2}$ **E)** $3-2\sqrt{2}$

9) We extent the sides of the triangle ABC so that $\Delta A = A\Gamma = \Gamma H$, $\Theta B = AB = AE$ and $KB = B\Gamma = \Gamma Z$. If the area of ABC is 1, what is the area of the hexagon ΔEZHΘK;



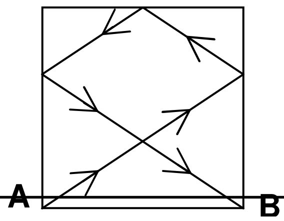
A) 9 **B)** 10
C) 12 **D)** 13
E) cannot be found

10) What is the value of the expression $\dfrac{22+33+44+55+66+77+88+99+1010}{2+3+4+5+6+7+8+9+10}$;

A) 10 **B)** 11 **C)** 101 **D)** $\dfrac{83}{3}$ **E)** other answer

4 point questions:

11) In a room of square shape and side 2 m, we place mirrors on the walls. A light ray begins at the vertex A and after three reflections, as shown in the figure, it ends at vertex B. How many metres is the length of the path of the ray?

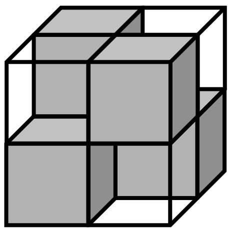


A) 7 **B)** $2\sqrt{13}$ **C)** 8 **D)** $4\sqrt{3}$ **E)** $2\left(\sqrt{2}+\sqrt{3}\right)$

12) 2009 kangaroos, each of them either light or dark, compare their heights. It is known that one light kangaroo is higher than exactly 8 dark kangaroos, one light kangaroo is higher than exactly 9 dark kangaroos, one light kangaroo is higher than exactly 10 dark kangaroos, and so on, and exactly one light kangaroo is higher than all dark kangaroos. What is the number of light kangaroos?

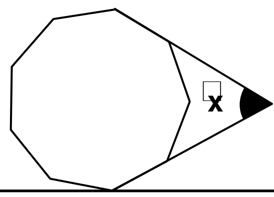
A) 1000 **B)** 1001 **C)** 1002 **D)** 1003 **E)** this situation is impossible

13) A cube measuring 2 x 2 x 2 is formed of four 1 x 1 x 1 white transparent and four 1 x 1 x 1 black non-transparent cubes (picture). They are placed in the way that the whole big cube is non-transparent, meaning that it is not possible to see through it neither from top to bottom, nor from front to back and not even from left to right. At least how many black cubes would we have to put into the big cube measuring 3 χ 3 χ 3 to make the whole cube non-transparent?



A) 6 **B)** 9 **C)** 10 **D)** 12 **E)** 18

14) In the figure we see a regular polygon with 9 sides. How many degrees is the angle x?

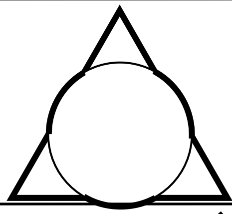


A) 40° **B)** 45° **C)** 50°
D) 55° **E)** 60°

15) What is the last digit of the number $1^2 - 2^2 + 3^2 - 4^2 + \ldots - 2008^2 + 2009^2$;

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

16) We overlap an equilateral triangle of side length 3 and a circle of radius 1 matching the centers of the two figures (point of intersection of perpendicular bisectors of the triangle). What is the length of the perimeter of the figure that we get?



A) $3+2\pi$ **B)** $6+\pi$ **C)** $9+\frac{\pi}{3}$ **D)** 3π **E)** $9+\pi$

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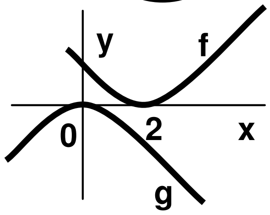
17) In the figure we see the graphs of two functions f and g. What is the relation between f and g?



A) $g(x)=f(x+2)$ **B)** $g(x-2)=-f(x)$ **C)** $g(x)=-f(-x+2)$
D) $g(-x)=-f(-x+2)$ **E)** $g(2-x)=-f(x)$

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18) Four problems were proposed to each of 100 contestants of a Mathematical Olympiad. 90 contestants solved the first problem, 85 contestants solved the second problem, 80 contestants solved the third problem, and 70 contestants solved the fourth problem. What is the smallest possible number of the contestants which solved all four problems?

A) 10 **B)** 15 **C)** 20 **D)** 25 **E)** 30

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19) What is the smallest natural number N with the property that the number $(2^2-1)\cdot(3^2-1)\cdot(4^2-1)\cdot\ldots\cdot(N^2-1)$ is a perfect square of a natural number?

A) 6 **B)** 8 **C)** 16 **D)** 27 **E)** other answer

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20) In the figure we have a 3×3 square, which is completed by real numbers such that the sum of the numbers in each of the columns, in each of the rows and in each of the diagonals is the same. Some of the numbers were erased but two of them are evident. What is the number is position a?

a		
		10
	20	

A) 5 **B)** 10 **C)** 15 **D)** 25 **E)** none of the previous

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5 point questions:

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21) Two runners A and B are going round a stadium. Each of them are running all the time at the same speed. A runs faster than B and it takes 3 minutes to A for one turn. A and B start together and 8 minutes further, A catches B up for the first time. How long does it take to B for one turn?

A) 6 min **B)** 8 min **C)** 4 min 30 sec **D)** 4 min 48 sec **E)** 4 min 20 sec

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22) Let N be the number of all 8-digit numbers with 8 different digits, none of which is 0. From these numbers lets say that M is the number of those that, in addition, the happen to be multiple of 9. Then

A) $M=\frac{N}{8}$ **B)** $M=\frac{N}{3}$ **C)** $M=\frac{N}{9}$ **D)** $M=\frac{8N}{9}$ **E)** $M=\frac{7N}{8}$

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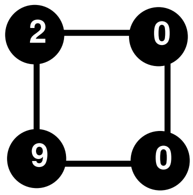
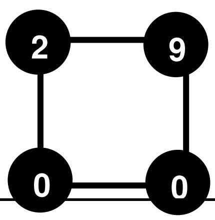
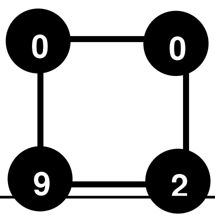
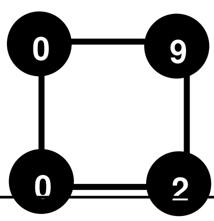
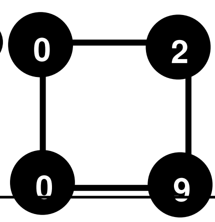
23) We write the numbers 2, 0, 0, 9 at the vertices of a square, as shown. In each move we choose one side of a square and either we add one unit in both number of the side or we subtract one unit from both numbers of the side. Which of the following squares we can never get by repeating the process as many times as needed?



A)


B)


C)


D)


E) we can get all four squares

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24) For how many natural numbers $N \geq 3$ there exist N sided polygons (not regular) whose angles are of the form θ, 2θ, 3θ, …, Nθ (for some $\theta > 0$) and they are all smaller than 180 degrees?

A) 1 **B)** 2 **C)** 3 **D)** 5 **E)** more than 5

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25) Eight papers numbered 1 through 8 were divided into two boxes A and B. The sum of the numbers in box A is the same with the sum of numbers in box B. Box A has in it 3 of the 8 papers. Which one of the following statements is for sure correct?

A) three of the papers in box B have odd numbers written on them
B) four of the papers in box B have even number written on them
C) the paper with number 1 is not in box B
D) the paper with the number 2 is in box B
E) the paper with the number 5 is in box B

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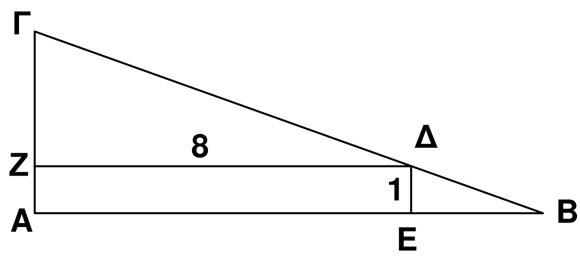
26) In a right triangle ΑΒΓ the point Δ of the hypotenuse is at distance 1 and 8 fro the vertical sides. If $\Delta E \cdot AB = \Gamma Z \cdot A\Gamma$, what is the length of AB;



A) $8+2\sqrt{2}$ **B)** $11-\sqrt{2}$ **C)** 10
D) $8+3\sqrt{2}$ **E)** $11+\dfrac{\sqrt{2}}{2}$

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27) If α, β, γ non-zero real numbers with $\alpha+\beta \neq 0, \beta+\gamma \neq 0, \gamma+\alpha \neq 0$, how many possible values could k have if $\kappa = \dfrac{\alpha}{\beta+\gamma} = \dfrac{\beta}{\gamma+\alpha} = \dfrac{\gamma}{\alpha+\beta}$;

A) 1 **B)** 2 **C)** 3 **D)** 4 **E)** 6

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28) We divide the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in N groups based on the following constraints

- each one of the numbers is located is exactly one of the N groups.
- each group contains at least two of the numbers.
- if two of the numbers are located in the same group then their sum is not a multiple of 3.

What is the smallest N that has this property?

A) 2 **B)** 3 **C)** 4 **D)** 5 **E)** 6

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29) In a group of clowns, the left shoe of each male is two sizes larger than the right show, while the left shoe of each female is one size larger than the right shoe. In order to save money, the group of the clowns purchased together some regular pairs of shoes. When each member of the group took the pair that s/he needed, two shoes were left. One was of size 45 and the other of size 36. What is the smallest possible number of clowns in the group?

A) 5 **B)** 6 **C)** 7 **D)** 8 **E)** 9

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30) We write on the board in a sequence the numbers 1, 3, 4, 7, 11, 18, …. From the third and beyond, every next number is equal to the sum of the two previous ones. What is the remainder of the one thousand number in the sequence when it is divided by 5?

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4
