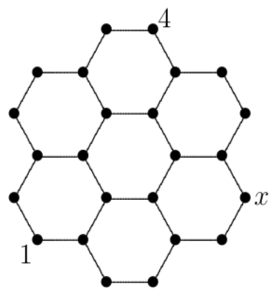
MATHEMATICS

LEVEL 11 – 12
(Β΄ - Γ΄ Λυκείου)

19 March 2011
10:00-11:15

3 point

1) In the next picture there should be a number at each of the dots • in such a way that the sum of the numbers at the ends of each segment is the same. Two of the numbers are already there. What goes in the place of x?



(A) 1 (B) 3 (C) 4 (D) 5 (E) 24

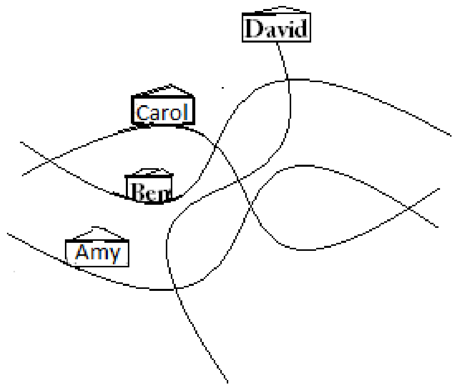
2) Three racers participated in a Formula -1 race: Michael, Fernando and Sebastian. Right after the start Michael was the first, Fernando – the second, Sebastian – the third. During the course of the race Michael and Fernando overtook each other 9 times, Fernando and Sebastian – 10 times, and Michael and Sebastian – 11 times. In what order did the racers finish?

(A) Michael, Fernando, Sebastian (B) Fernando, Sebastian, Michael (C) Sebastian, Michael, Fernando (D) Sebastian, Fernando, Michael (E) Fernando, Michael, Sebastian

3) If $2^x = 15,\ 15^y = 32$ then xy is equal to…

(A) 5 (B) $\log_2 15 + \log_{15} 32$ (C) $\log_2 47$ (D) 7 (E) $\sqrt{47}$

4) Jane, who is not very good at drawing correctly, tried to sketch a map of her home village. She managed to draw the four streets, their seven crossings and the houses of her friends, but in reality Arrow Street, Nail Street and Ruler Street are all straight. The fourth street is Curvy Road. Who lives on Curvy Road?

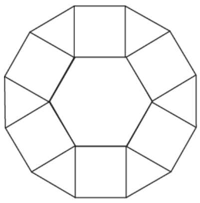


(A) Amy (B) Ben (C) Carol (D) David (E) We cannot decide from Jane's sketch.

5) All 4-digit numbers the sum of whose digits is 4 are written in descending order. In which place in this sequence is the number 2011 situated?

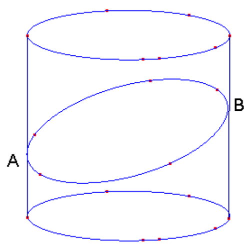
(A) 6th (B) 7th (C) 8th (D) 9th (E) 10th

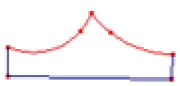
6) We are given a regular hexagon of side 1, 6 squares and 6 (equilateral) triangles as shown. What is the perimeter of the figure?

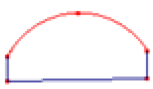


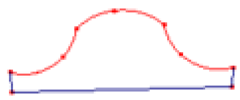
(A) $6(1+\sqrt{2})$ (B) $6\left(1+\frac{\sqrt{3}}{2}\right)$ (C) 9 (D) $6+3\sqrt{2}$ (E) 12

7) A rectangular piece of paper is wrapped around a cylinder and a plane cut through the points A and B is made trough the cylinder as shown. The bottom part of the paper is then unwrapped. Which picture coud be the result?



(A)


(B)


(C)


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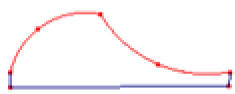
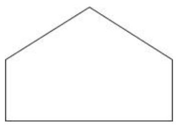
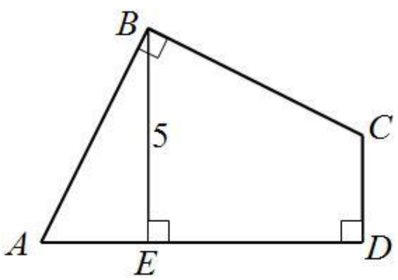


(E)


8) Find the area of quadrilateral ABCD (see the figure) such, that $AB = BC$, $\angle ABC = \angle ADC = 90°$, $BE \perp AD$, $BE = 5$



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(A) 20 (B) 22,5 (C) 25 (D) 27,5 (E) 30

9) Andrew wrote the odd numbers from 1 to 2011 on a board and then Bob erased all multiples of 3. How many numbers were left on the board?

(A) 335 (B) 336 (C) 671 (D) 1005 (E) 1006

10) Max and Hugo throw a handful of dices to decide who shall be the first to jump into a cold lake. If there are no sixes it will be Max. If there is one six it will be Hugo and if there are more sixes they will not take a swim that day. How many dice should they throw if they want the risk of having to jump in first to be equally divided between the two of them?

(A) 3 (B) 5 (C) 8 (D) 9 (E) 17

4 point

11) A rectangle was sectioned into three rectangles. One of them has size 7 by 11. Another one has size 4 by 8. Find the size of the third rectangle with the maximal area.

(A) 1 by 11 (B) 3 by 4 (C) 3 by 8 (D) 7 by 8 (E) 7 by 11

12) Mike wants to write integers in the cells of the 3×3 table so that the sum of the numbers in each 2×2 square equals 10. Four numbers are already written in the table as it is shown in the figure. Which of the following values could be the sum of other five?

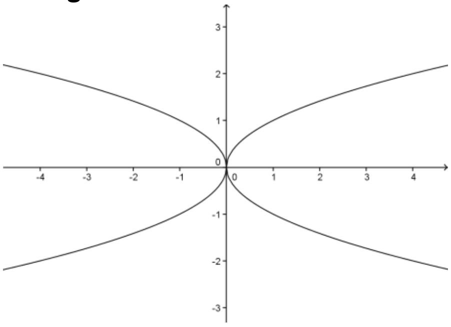
	2	
1		3
	4	

(A) 9 (B) 10 (C) 12 (D) 13 (E) None of these is possible

13) 48 children went on a ski trip. Six of them had exactly one sibling on the trip, nine children went with exactly two siblings and four of them with exactly three siblings. The rest of the children didn´t have any siblings on the trip. How many families went on this trip?

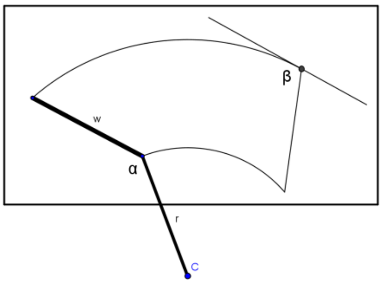
(A) 19 (B) 25 (C) 31 (D) 36 (E) 48

14) How many of the graphs of functions
$y = x^2, y = -x^2, y = +\sqrt{x}, y = -\sqrt{x}, y = +\sqrt{-x}, y = -\sqrt{-x}, y = +\sqrt{|x|}, y = -\sqrt{|x|}$ **are included in the figure below? The horizontal axes is X and the vertical is Y.**



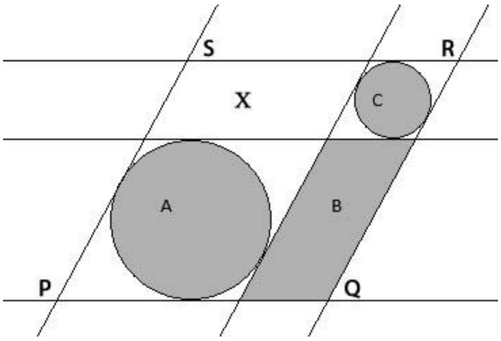
(A) None (B) 2 (C) 4 (D) 6 (E) All, 8

15) The rear windshield wiper of a car is constructed in such a way that the wiper blade w and the connecting rod r are of equal lengths and are joined at an angle α. The wiper pivots on the center C and clears the area as shown. Determine the angle β between the right-hand edge of the cleared area and the tangent of the curved upper edge.



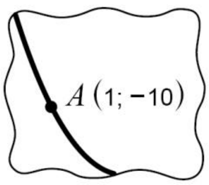
(A) $\frac{3\pi-\alpha}{2}$ (B) $\pi-\frac{\alpha}{2}$ (C) $\frac{3\pi}{2}-\alpha$ (D) $\frac{\pi}{2}+\alpha$ (E) $\pi+\frac{\alpha}{2}$

16) We have three horizontal lines and three mutually parallel slanting lines. Both circles are tangent to four of the lines. A, B and C are areas of the shaded figures. D is the area of the parallelogram PQRS. What is the smallest number of areas A, B, C and D that must be known to be able to calculate the area of parallelogram X?



(A) 1 (B) 2 (C) 3 (D) 4 (E) X cannot be calculated from A, B, C and D

17) On the (x,y)-plane with axes positioned in a standard way, the point A(1; – 10) was marked on the parabola $y=ax^2+bx+c$ **. After that, the coordinate axes and almost all of the parabola were erased. Which of the statements below can be false?**



(A) $a>0$ (B) $b<0$ (C) $a+b+c<0$ (D) $b^2>4ac$ (E) $c<0$

18) The sides AB, BC, CD, DE, EF and FA of a hexagon are all tangent to a common circle. The lengths of the sides AB, BC, CD, DE and EF are 4, 5, 6, 7 and 8 respectively. Then the length of side FA is…

(A) 9 (B) 8 (C) 7 (D) 6 (E) The length cannot be calculated from this information

19) Find the sum of all positive integers x smaller than 100 such that $x^2 - 81$ is a multiple of 100.

(A) 200 (B) 100 (C) 90 (D) 81 (E) 50

20) The brothers Andrej and Brano gave truthful answers to a question about how many members their chess club has. Andrej said: „All the members of our club, except for five of them, are boys.“ Brano: „In every group of six members there are necessarily at least four girls.“ How many members does their chess club have?

(A) 6 (B) 7 (C) 8 (D) 12 (E) 18

5 point

21) There are balls in a raffle bucket. One positive integer is written on each ball, a different number on each. A number divisible by 6 is written on 30 balls, a number divisible by 7 is written on 20 balls and a number divisible by 42 is written on 10 balls. At least how many balls must be in the bucket?

(A) 30 (B) 40 (C) 53 (D) 54 (E) 60

22) Consider the two arithmetic sequences 5, 20, 35, … and 35, 61, 87,… . How many different arithmetic sequences of positive integers exist having both of them as a subsequence?

(A) 1 (B) 3 (C) 5 (D) 26 (E) Infinitely many

23) The sequence of numerical functions $f_1(x), f_2(x), \ldots$ satisfies the following conditions (1) $f_1(x)=x$; (2) $f_{n+1}(x)=\dfrac{1}{1-f_n(x)}$. Determine the value of $f_{2011}(2011)$.

(A) 2011 (B) $-\dfrac{1}{2010}$ (C) $\dfrac{2010}{2011}$ (D) 1 (E) – 2011

24) A box contains some red balls and some green balls. If we choose randomly two balls from the box, they are of the same colour with probability 1/2. Which of the following could be the total number of the balls in the box?

(A) 81 (B) 101 (C) 1000 (D) 2011 (E) 10001

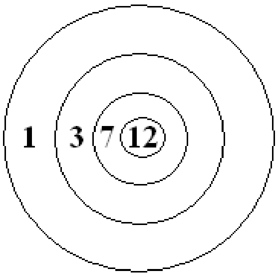
25) **An airline company doesn´t charge luggage fees if the luggage is under a certain weight limit. For every extra kilogram a fee is charged. The luggage of Mr. and Mrs. Trip weighed 60 kg and they paid 3 €. Mr. Wander´s luggage weighed the same but he paid 10,50 €. What is the maximum weight of luggage one passenger doesn`t have to pay for?**

(A) 10 (B) 18 (C) 20 (D) 25 (E) 39

26) **What is the smallest positive integer value of the expression** $\frac{K \cdot A \cdot N \cdot G \cdot A \cdot R \cdot O \cdot O}{G \cdot A \cdot M \cdot E}$ **(different letters stand for different nonzero positive digits while equal letters stand for equal digits).**

(A) 1 (B) 2 (C) 3 (D) 5 (E) 7

27) **Robin Hood shoots three arrows at a target, earning points for each shot as shown in the figure. How many different point totals can he obtain in this way?**



(A) 13 (B) 17 (C) 19 (D) 20 (E) 21

28) **Let** a, b **and** c **be positive integers such that** $a^2 = 2b^3 = 3c^5$**. What is the minimum number of divisors of** abc **(including 1 and** abc**)?**

(A) 30 (B) 49 (C) 60 (D) 77 (E) 1596

29) **Twenty different positive integers are written in a** 4×5 **table. Any two neighbours (numbers in cells with a common side) have a common divisor greater than 1. If** n **is the biggest number in the table find the least possible value of** n**.**

(A) 21 (B) 24 (C) 26 (D) 27 (E) 40

30) **A 3 × 3 × 3 cube is composed of 27 identical small cubes. A plane is perpendicular to a diagonal of the large cube and passes through its centre. How many small cubes does that plane intersect?**

(A) 17 (B) 18 (C) 19 (D) 20 (E) 21