MATHEMATICS

LEVEL 9 – 10
(Γ΄ Γυμνασίου- Α΄ Λυκείου)

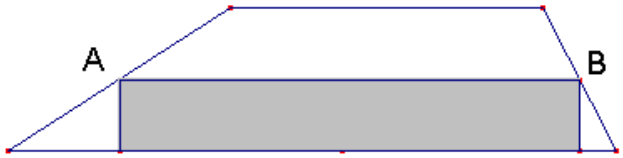
19 March 2011
10:00-11:15

3 point

1) A zebra crossing has white and black stripes, all of breadth 50 cm. On a road the crossing starts and ends with a white stripe. The crossing has 8 white stripes. What is the breadth of the road?

(A) 7 m (B) 7,5 m (C) 8 m (D) 8,5 m (E) 9 m

2) The rectangle has an area 13 cm^2. A and B are the midpoints of the sides of the trapezoid. What is the area of the trapezoid?

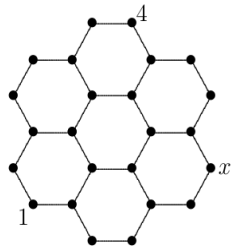


(A) 24 (B) 25 (C) 26 (D) 27 (E) 28

3) Given the expressions, $S_1 = 2\cdot3+3\cdot4+4\cdot5$, $S_2 = 2^2+3^2+4^2$, $S_3 = 1\cdot2+2\cdot3+3\cdot4$, which of the following relationships are true?

(A) $S_2 < S_1 < S_3$ (B) $S_1 < S_2 = S_3$ (C) $S_1 < S_2 < S_3$ (D) $S_3 < S_2 < S_1$ (E) $S_2 = S_2 < S_3$

4) In the next picture there should be a number at each of the dots • in such a way that the sum of the ends of each segment is the same. Two of the numbers are already there. What goes in the place of x?



(A) 1 (B) 3 (C) 4 (D) 5 (E) not enough information

5) When 2011 was divided by a certain number, the remainder was 1011. Which of the given numbers was the divisor?

(A) 100 (B) 500 (C) 1000
(D) some other number (E) it is not possible to get this remainder

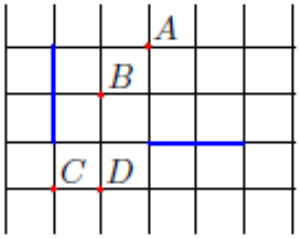
6) A rectangular mosaic with area 360 cm² is made from square tiles, all the same size. The mosaic is 24 cm lenght and 5 tiles wide. What is the area of each tile in cm²?

(A) 1 (B) 4 (C) 9 (D) 16 (E) 25

7) All 4-digit numbers whose sum of digits is 4 are written in descending order. In which place in this sequence is the number 2011 situated?

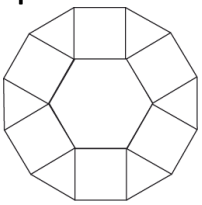
(A) 6th (B) 7th (C) 8th (D) 9th (E) 10th

8) Each of the given two segments shown in darker colour is a rotation image of the one another . Which of points shown could be centres of such rotations?



(A) Only A (B) A and C. (C) A and D. (D) Only D. (E) A,B,C and D.

9) The diagram shows a shape consisting of a regular hexagon of side one unit, six triangles and six squares.What is the perimeter of the shape?



(A) $6(1+\sqrt{2})$ (B) $6\left(1+\frac{\sqrt{3}}{2}\right)$ (C) 12 (D) $6+3\sqrt{2})$ (E) 9

10) Three normal dice are piled on top of each other so that the sum of spots on the two faces that meet are always 5. One of the visible faces on the bottom dice shows one spot. How many spots does the top face on the top dice show?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

4 Points

11) In a certain month there were 5 Mondays, 5 Tuesdays, and 5 Wednesdays. In the preceding month there were only 4 Sundays. Which of the following will the next month definitely include?

(A) exactly 4 Fridays (B) exactly 4 Saturdays (C) 5 Sundays (D) 5 Wednesdays (E) this situation is impossible

12) Three sportsmen participated in a race: Michael, Fernando and Sebastian. Right after the start Michael was the first, Fernando second, and Sebastian third. During the race Michael and Fernando overtook each other 9 times, Fernando and Sebastian 10 times, and Michael and Sebastian 11 times. In what order did they finish?

(A) Michael, Fernando, Sebastian (B) Fernando, Sebastian, Michael (C) Sebastian, Michael, Fernando (D) Sebastian, Fernando, Michael (E) Fernando, Michael, Sebastian

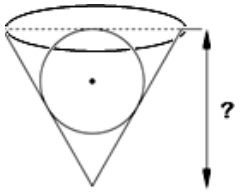
13) Given that $9^n+9^n+9^n=3^{2011}$, what is the value of n?

(A) 1005 (B) 1006 (C) 2010 (D) 2011 (E) none of them

14) I have two cubes with the sides of length *a* cm and *a +1* cm. The big cube is full of water and the small one is empty. I pour some water from the big cube into the small cube till this one is full, leaving 217 lt in the big cube. How much water was poured into the small cube?

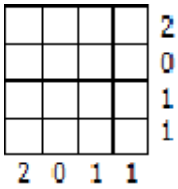
(A) 243 lt (B) 512 lt (C) 125 lt (D) 1331 lt (E) 729 lt

15) A marble with radius 15 is rolled in a conical hole and fits exactly. Viewed from the side the conical hole is an equilateral triangle. How deep is the hole?



(A) $30\sqrt{2}$ (B) $25\sqrt{3}$ (C) 45 (D) 60 (E) $60(\sqrt{3}-1)$

16) The cells of this 4x4-grid will be coloured black or white. Next to the rows and columns of this grid is indicated the number of cells in that column/row that must be black. In how many ways can this be done?



(A) 0 (B) 1 (C) 3 (D) 5 (E) 9

17) What is the maximal number of consecutive 3-digit numbers that have at least one odd digit?

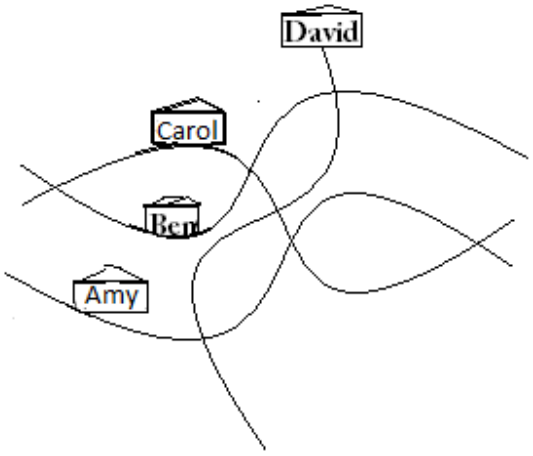
(A) 1 (B) 10 (C) 110 (D) 111 (E) 221

18) Nick wants to write integers in cells of the 3×3 table so that the sums of the numbers in each 2×2 square equal 10. He has already written five numbers in the table as it is shown in the figure. Find the sum of the other four numbers.

1		0
	2	
4		3

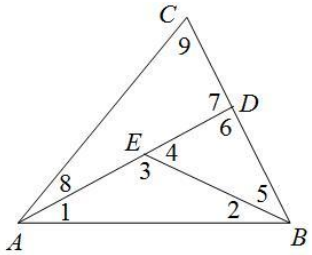
(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

19) During a bumpy sailing, Jane tried to sketch a map of her home village. She managed to draw the four streets, their seven crossings and the houses of her friends, but in reality Arrow Street, Nail Street and Ruler Street are all straight. The fourth street is Curvy Road. Who lives on Curvy Road?



(A) Amy (B) Ben (C) Carol (D) David (E) You need a better map to be able to tell.

20) In triangle ABC, a point D is chosen on the side BC, then point E is chosen on the segment AD. We thus obtain 9 angles denoted in the figure by the numbers 1, 2, …, 9. Find the minimum possible number of different values that the angles 1, 2, …, 9 could take.



(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

5 point

21) Simon had a glass cube with the edge 1 dm long. He stuck several congruent golden squares on the cube so that the cube looked the same from all the sides (see the picture). How many cm^2 are golden?

(A) 37.5 (B) 150 (C) 225 (D) 300 (E) 375

22) Call the five-digit number $\overline{abcde}$ *interesting* if its digits are distinct and $a=b+c+d+e$. How many interesting numbers are there?

(A) 72 (B) 144 (C) 168 (D) 216 (E) 288

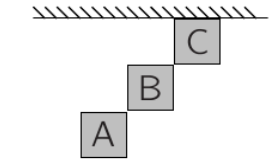
23) The number x and y are both greater than 1. Which of the following fractions has the greatest value?

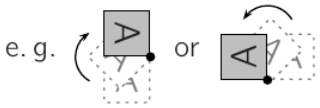
(A) $\frac{x}{y+1}$ (B) $\frac{x}{y-1}$ (C) $\frac{2x}{2y+1}$ (D) $\frac{2x}{2y-1}$ (E) $\frac{3x}{3y+1}$

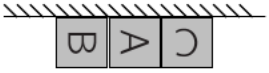
24) A regular tetrahedron ABCD has its face ABC in the plane ε. The edge BC is on the line s. A different regular tetrahedron BCDE shares a face with ABCD. Where does the line DE intersect with ε ?

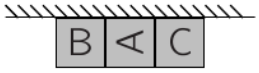
(A) on the same side of s as A, inside ABC
(B) on the same side of s as A, outside ABC
(C) on the opposite side of s relative to A
(D) DE is parallel to ε
(E) The answer depends on the edge lengths of the tetrahedra.

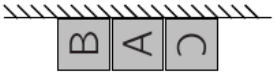
25) **Three big boxes were delivered to a warehouse and put on the floor as shown from above in the top picture. The boxes have to be placed neatly along the wall in a certain order. They are so heavy that they can only be rotated around one of the bottom corners in a 90 degree angle (see examples in the bottom picture). Which picture is possible?**

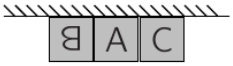




(A) 

(B) 

(C) 

(D) 

(E) all four pictures are possible

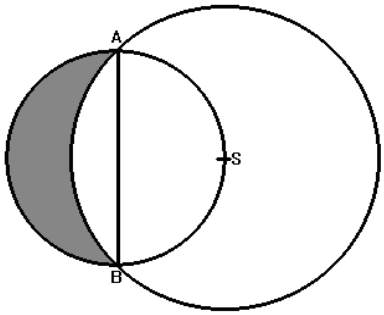
26) **How many ordered pairs of natural numbers (x, y) satisfy the equation** $\frac{1}{x}+\frac{1}{y}=\frac{1}{3}$ **?**

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

27) **For an integer** $n \geq 2$ **denote by** $\langle n \rangle$ **the biggest prime number, which does not exceed** n**. How many positive integers** k **satisfy the equation** $\langle k+1 \rangle + \langle k+2 \rangle = \langle 2k+3 \rangle$ **?**

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

28) **Two circles are constructed as shown in the figure. The segment** AB **is the diameter of the smaller circle. The center** S **of the greater circle lies on the smaller circle, the radius of the greater circle is** r**. What is the area of the shaded region?**



(A) $\frac{\pi}{6}\cdot r^2$ (B) $\frac{\sqrt{3}\cdot\pi}{12}\cdot r^2$ (C) $\frac{1}{2}\cdot r^2$ (D) $\frac{\sqrt{3}}{4}\cdot r^2$ (E) another answer

29) **How many quadruples of edges of a cube possess the property that any two edges in such a quadruple have no common vertices?**

(A) 6 (B) 8 (C) 9 (D) 12 (E) 18

30) **Find all** n **(0 <** n **< 9) such that it is possible to mark some cells in a 5×5 square so that there are exactly** n **marked cells in each 3×3 square?**

(A) 1 (B) 1 and 2 (C) 1, 2 and 3 (D) 1, 2, 7 and 8 (E) all possible